5.2 Properties of Determinants

520. The value of a determinant remains unchanged if rows are changed to columns and columns to rows.

$$\begin{vmatrix} \mathbf{a}_1 & \mathbf{a}_2 \\ \mathbf{b}_1 & \mathbf{b}_2 \end{vmatrix} = \begin{vmatrix} \mathbf{a}_1 & \mathbf{b}_1 \\ \mathbf{a}_2 & \mathbf{b}_2 \end{vmatrix}$$

521. If two rows (or two columns) are interchanged, the sign of the determinant is changed.

$$\begin{vmatrix} \mathbf{a}_1 & \mathbf{b}_1 \\ \mathbf{a}_2 & \mathbf{b}_2 \end{vmatrix} = - \begin{vmatrix} \mathbf{a}_2 & \mathbf{b}_2 \\ \mathbf{a}_1 & \mathbf{b}_1 \end{vmatrix}$$

522. If two rows (or two columns) are identical, the value of the determinant is zero.

$$\begin{vmatrix} a_1 & a_1 \\ a_2 & a_2 \end{vmatrix} = 0$$

523. If the elements of any row (or column) are multiplied by a common factor, the determinant is multiplied by that factor.

$$\begin{vmatrix} ka_1 & kb_1 \\ a_2 & b_2 \end{vmatrix} = k \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

524. If the elements of any row (or column) are increased (or decreased) by equal multiples of the corresponding elements of any other row (or column), the value of the determinant is unchanged.

$$\begin{vmatrix} a_1 + kb_1 & b_1 \\ a_2 + kb_2 & b_2 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

